4.6 Applying the Exponent Laws

LESSON FOCUS Apply the exponent laws to simplify expressions.

Make Connections

Recall the exponent laws for integer bases and whole number exponents.

Product of powers:	$a^m \cdot a^n = a^{m+n}$
Quotient of powers:	$a^m \div a^n = a^{m-n}$, $a \neq 0$
Power of a power:	$(a^m)^n = a^{mn}$
Power of a product:	$(ab)^m = a^m b^m$
Power of a quotient:	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, $b \neq 0$

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What other types of numbers could be a base? An exponent?
How would you use the exponent laws to evaluate an expression with these numbers?

1. Product of powers: $a^m \cdot a^n = a^{m+n}$
2. Quotient of powers: $a^m \div a^n = a^{m-n}$, $a \neq 0$
3. Power of a power: $(a^m)^n = a^{mn}$
4. Power of a product: $(ab)^m = a^m b^m$
5. Power of a quotient: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, $b \neq 0$

Ex:

#1) $2^3 \cdot 2^5 = 2^{3+5} = 2^8$

#2) $\frac{a^5}{a^{-3}} = a^{5-(-3)} = a^8$

#3 $(3^2)^{\frac{2}{3}} = 3^{2 \times \frac{2}{3}} = 3^{4/3}$

#4 $(a^2 b^3)^4 = a^{2\times4} b^{3\times4} = a^8 b^{12}$

#5 $\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2}$

Please try all of the following:

Q Identify and explain the errors in the following:

1st set (whole number exponents):

#1 a) $4^3 + 4^2 = 4^5$ b) $\frac{x^6}{x^3} = x^2$ c) $(10^2)^5 = 10^7$ d) $\left(\frac{1}{4}\right)^2 = \frac{1}{8}$

e) $(x-y)^3 = 3x - 3y$ f) $3^5 \times 3^2 = 3^{10}$ g) $5^3 \div 5^4 = \frac{3}{4}$

2nd set (integral exponents):

#2 a) $a^4 \bullet a^{-2} = a^{-8}$ b) $b^{-10} \div b^5 = b^{-5}$ c) $(c^{-3})^2 = c^{-1}$ d) $\left(\frac{2}{3}\right) - 2 = \frac{-4}{-6}$

3rd set (rational exponents):

#3 a) $2^{\frac{1}{2}} \bullet 2^{\frac{1}{2}} = 2^{\frac{1}{4}}$ b) $3^{\frac{3}{4}} \div 3^{\frac{1}{4}} = 3^3$ c) $\left(4^{\frac{2}{5}}\right)^2 = 4^{\frac{2}{25}}$ d) $\left(5^{\frac{1}{2}}\right)^2 = 5^{\frac{1}{2^2}}$

Q Fill in the blanks:

#4 a) $5^{-2} = \frac{\square}{\square}$ b) $6^{\square} = \frac{1}{6^2}$ c) $\square^{-6} = \frac{1}{10^6}$ d) $4^{-x} = \frac{1}{\square}$

Q Solve the following, by substituting the values given:

#5 a) $5x^4 + 6xy \quad if \quad x = 2, \; y = 3$

b) $(2x)^2 \quad if \; x = 4$

c) $(t+s)^{-3} \quad if \; t = 2, \; s = 4$

Show your work

Q Indicate if the following statements are always true, sometimes true, or false. Justify your answer.

a) The value of a power with a negative exponent is less than 0.

b) The value of a power for which the base is a fraction is less than 1.

c) Two powers with the exponent 0 have the same value.

Q During an exam, three students evaluate $2^{-2} \times 2^0$ as follows:

Thomas: $2^{-2} \times 2^0 = 4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

Sean: $2^{-2} \times 2^0 = 2^0 = 1$

Michel: $2^{-2} \times 2^0 = 4^0 = 1$

a) Identify the errors made by the students.

b) What is the correct answer? Justify your answer by explaining each step.

Act In pairs explain how to evaluate powers such as $(-3)^{-2}$ and -3^{-2}. Compare your answers with other groups, and then as a class.

THINK ABOUT IT

Work on your own.

What is the value of $\left(\frac{a^6b^9}{a^5b^8}\right)^{-2}$ when $a = -3$ and $b = 2$?

Compare strategies with a classmate.
If you used the same strategy, find a different strategy.
Which strategy is more efficient, and why?

$\left(\frac{a^6b^9}{a^5b^8}\right)^{-2}$

$a = -3$ and $b = 2$?

Simplify by writing as a single power. Explain the reasoning.

a) $0.3^{-3} \cdot 0.3^{5}$

$= 0.3^{2}$

b) $\left[\left(-\frac{3}{2}\right)^{-4}\right]^{2} \cdot \left[\left(-\frac{3}{2}\right)^{2}\right]^{3}$

$\left(-\frac{3}{2}\right)^{-8} \cdot \left(-\frac{3}{2}\right)^{6} = \left(-\frac{3}{2}\right)^{-2}$

$= \left(-\frac{2}{3}\right)^{2}$

c) $\frac{(1.4^3)(1.4^4)}{1.4^{-2}} =$

$3+4-(-2)$

$= 1.4^{9}$

d) $\left(\frac{7^{\frac{2}{3}}}{7^{\frac{1}{3}} \cdot 7^{\frac{5}{3}}}\right)^{6}$

CHECK YOUR UNDERSTANDING

SOLUTION

Example 1 Simplifying Numerical Expressions with Rational Number Bases

Simplify by writing as a single power. Explain the reasoning.

a) $0.3^{-3} \cdot 0.3^{5}$

b) $\left[\left(-\frac{3}{2}\right)^{-4}\right]^{2} \cdot \left[\left(-\frac{3}{2}\right)^{2}\right]^{3}$

c) $\frac{(1.4^{3})(1.4^{4})}{1.4^{-2}}$

d) $\left(\frac{7^{\frac{2}{3}}}{7^{\frac{1}{3}} \cdot 7^{\frac{5}{3}}}\right)^{6}$

SOLUTION

a) $0.3^{-3} \cdot 0.3^{5}$

Use the product of powers law:

When the bases are the same, add the exponents.

$0.3^{-3} \cdot 0.3^{5} = 0.3^{(-3)+5}$

$= 0.3^{2}$

(Solution continues.)

Example 1 Simplifying Numerical Expressions with Rational Number Bases

b) $\left[\left(-\frac{3}{2}\right)^{-4}\right]^{2} \cdot \left[\left(-\frac{3}{2}\right)^{2}\right]^{3}$

First use the power of a power law:

For each power, multiply the exponents.

$\left[\left(-\frac{3}{2}\right)^{-4}\right]^{2} \cdot \left[\left(-\frac{3}{2}\right)^{2}\right]^{3} = \left(-\frac{3}{2}\right)^{(-4)(2)} \cdot \left(-\frac{3}{2}\right)^{(2)(3)}$

Then use the product of powers law.

$\left[\left(-\frac{3}{2}\right)^{-4}\right]^{2} \cdot \left[\left(-\frac{3}{2}\right)^{2}\right]^{3} = \left(-\frac{3}{2}\right)^{-8} \cdot \left(-\frac{3}{2}\right)^{6}$

$= \left(-\frac{3}{2}\right)^{-2}$ Write with a positive exponent.

$= \left(-\frac{2}{3}\right)^{2}$

(Solution continues.)

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Example 1 Simplifying Numerical Expressions with Rational Number Bases

c) $\dfrac{(1.4^3)(1.4^4)}{1.4^{-2}}$ Use the product of powers law.

$= \dfrac{1.4^{3+4}}{1.4^{-2}}$

$= \dfrac{1.4^7}{1.4^{-2}}$ Use the quotient of powers law.

$= 1.4^{7-(-2)}$

$= 1.4^9$

d) $\left(\dfrac{7^{\frac{2}{3}}}{7^{\frac{1}{3}} \cdot 7^{\frac{5}{3}}}\right)^6$ Use the product of powers law.

$= \left(\dfrac{7^{\frac{2}{3}}}{7^{\frac{1}{3}+\frac{5}{3}}}\right)^6$

$= \left(\dfrac{7^{\frac{2}{3}}}{7^{\frac{6}{3}}}\right)^6$ Use the quotient of powers law.

(Solution continues.)

Example 1 Simplifying Numerical Expressions with Rational Number Bases

$= \left(7^{\frac{2}{3}-\frac{6}{3}}\right)^6$

$= \left(7^{-\frac{4}{3}}\right)^6$ Use the power of a power law.

$= 7^{\left(-\frac{4}{3}\right)(6)}$

$= 7^{-\frac{24}{3}}$

$= 7^{-8}$ Write with a positive exponent.

$= \dfrac{1}{7^8}$

CHECK YOUR UNDERSTANDING

4.6 Applying the Exponent Laws

Example 2 Simplifying Algebraic Expressions with Integer Exponents

Simplify. Explain the reasoning.

a) $(x^3y^2)(x^2y^{-4})$ b) $\dfrac{10a^5b^3}{2a^2b^{-2}}$

SOLUTION

CHECK YOUR UNDERSTANDING

Example 2 Simplifying Algebraic Expressions with Integer Exponents

Simplify. Explain the reasoning.

a) $(x^3y^2)(x^2y^{-4})$ b) $\dfrac{10a^5b^3}{2a^2b^{-2}}$

SOLUTION

a) $(x^3y^2)(x^2y^{-4}) = x^3 \cdot y^2 \cdot x^2 \cdot y^{-4}$ x^3y^2 means $x^3 \cdot y^2$

$= x^3 \cdot x^2 \cdot y^2 \cdot y^{-4}$ Use the product of powers law.

$= x^{3+2} \cdot y^{2+(-4)}$

$= x^5 \cdot y^{-2}$ Write with a positive exponent.

$= x^5 \cdot \dfrac{1}{y^2}$

$= \dfrac{x^5}{y^2}$

b) $\dfrac{10a^5b^3}{2a^2b^{-2}} = \dfrac{10}{2} \cdot \dfrac{a^5}{a^2} \cdot \dfrac{b^3}{b^{-2}}$ Use the quotient of powers law.

$= 5 \cdot a^{5-2} \cdot b^{3-(-2)}$

$= 5 \cdot a^3 \cdot b^5$

$= 5a^3b^5$

CHECK YOUR UNDERSTANDING

4.6 Applying the Exponent Laws

Example 3 Simplifying Algebraic Expressions with Rational Exponents

Simplify. Explain the reasoning.

a) $(8a^3b^6)^{\frac{1}{3}}$

b) $(x^{\frac{3}{2}}y^2)(x^{\frac{1}{2}}y^{-1})$

c) $\dfrac{4a^{-2}b^{\frac{2}{3}}}{2a^2b^{\frac{1}{3}}}$

d) $\left(\dfrac{100a}{25a^5b^{-\frac{1}{2}}}\right)^{\frac{1}{2}}$

SOLUTION

CHECK YOUR UNDERSTANDING

Example 3 Simplifying Algebraic Expressions with Rational Exponents

Simplify. Explain the reasoning.

a) $(8a^3b^6)^{\frac{1}{3}}$

b) $(x^{\frac{3}{2}}y^2)(x^{\frac{1}{2}}y^{-1})$

c) $\dfrac{4a^{-2}b^{\frac{2}{3}}}{2a^2b^{\frac{1}{3}}}$

d) $\left(\dfrac{100a}{25a^5b^{-\frac{1}{2}}}\right)^{\frac{1}{2}}$

SOLUTION

a) $(8a^3b^6)^{\frac{1}{3}} = 8^{\frac{1}{3}} \cdot a^{3\left(\frac{1}{3}\right)} \cdot b^{6\left(\frac{1}{3}\right)}$ Using the power of a power law.

$= (2^3)^{\frac{1}{3}} \cdot a^1 \cdot b^2$

$= 2ab^2$

b) $(x^{\frac{3}{2}}y^2)(x^{\frac{1}{2}}y^{-1}) = x^{\frac{3}{2}} \cdot x^{\frac{1}{2}} \cdot y^2 \cdot y^{-1}$ Use the product of powers law.

$= x^{\frac{3}{2}+\frac{1}{2}} \cdot y^{2+(-1)}$

$= x^2y$

(Solution continues.)

Example 3 Simplifying Algebraic Expressions with Rational Exponents

c) $\dfrac{4a^{-2}b^{\frac{2}{3}}}{2a^{2}b^{\frac{1}{3}}} = \dfrac{4}{2} \cdot \dfrac{a^{-2}}{a^{2}} \cdot \dfrac{b^{\frac{2}{3}}}{b^{\frac{1}{3}}}$ Use the quotient of powers law.

$= 2 \cdot a^{(-2)-2} \cdot b^{\frac{2}{3}-\frac{1}{3}}$

$= 2 \cdot a^{-4} \cdot b^{\frac{1}{3}}$ Write with a positive exponent.

$= \dfrac{2b^{\frac{1}{3}}}{a^{4}}$

(Solution continues.)

Example 3 Simplifying Algebraic Expressions with Rational Exponents

d) $\left(\dfrac{100a}{25a^{5}b^{-\frac{1}{2}}}\right)^{\frac{1}{2}} = \left(\dfrac{100}{25} \cdot \dfrac{a^{1}}{a^{5}} \cdot \dfrac{1}{b^{-\frac{1}{2}}}\right)^{\frac{1}{2}}$ Simplify inside the brackets first. Use the quotient of powers law. Write with a positive exponent.

$= \left(4 \cdot a^{1-5} \cdot b^{\frac{1}{2}}\right)^{\frac{1}{2}}$

$= \left(4 \cdot a^{-4} \cdot b^{\frac{1}{2}}\right)^{\frac{1}{2}}$ Use the power of a power law.

$= 4^{\frac{1}{2}} \cdot a^{(-4)\left(\frac{1}{2}\right)} \cdot b^{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}$

$= 2 \cdot a^{-2} \cdot b^{\frac{1}{4}}$ Write with a positive exponent.

$= \dfrac{2b^{\frac{1}{4}}}{a^{2}}$

CHECK YOUR UNDERSTANDING

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